HYPER VERSION

Received: *March 8, 2000,* Accepted: *May 2, 2000*

Chiral symmetry breaking in the AdS/CFT correspondence

Jacques Distler and Frederic Zamora*

Theory Group, Physics Department
University of Texas at Austin
Austin TX 78712, USA
E-mail: distler@golem.ph.utexas.edu, zamora@zerbina.ph.utexas.edu

Abstract: We study the SU(3)-invariant relevant deformation of $D = 4$ $\mathcal{N} = 4$ SU(N) gauge theory at large N using the AdS/CFT correspondence. At low energies, we obtain a non-supersymmetric gauge theory with three left-handed quarks in the adjoint of SU(N). In terms of the five-dimensional gauged supergravity, there is an unstable critical point in the scalar potential for fluctuations of some fields in a non-trivial representation of the symmetry group SU(3). On the field theory side, this corresponds to dynamical breaking of the SU(3) chiral symmetry down to SO(3). We compute the condensate of the quark bilinear and the two-point correlation function of the spontaneously broken currents from supergravity and find a non-zero "pion" decay constant, f_π.

Keywords: Spontaneous Symmetry Breaking, String Duality, Supergravity Models.

*Work supported in part by NSF Grant PHY9511632 and the Robert A. Welch Foundation.

JHEP05(2000)005

Contents

1. Introduction

Consider SU(N) Yang-Mills fields, coupled to N_f Weyl fermions, λ^i ($i = 1, \ldots, N_f$), in the adjoint of the gauge group SU(N), often referred to as "adjoint QCD". For $N_f \leq 5$, the theory is asymptotically free and very plausibly confines for energies smaller than the dynamically-generated scale. The global symmetry group is SU(N_f). The Weyl fermions are in the fundamental representation, which is complex for $N_f > 2$. This chiral symmetry is presumably dynamically broken down to SO(N_f), its largest vector-like subgroup[1] [1, 2]. The order parameter for chiral symmetry breaking is the expectation value of the quark bilinear

$$\left\langle \mathrm{tr}\lambda^i\lambda^j \right\rangle \propto \delta^{ij}\,, \quad i,j = 1, \ldots, N_f\,. \tag{1.1}$$

[1] Actually, the chiral symmetry group is $(\mathrm{SU}(N_f) \times \mathbb{Z}_{2N_cN_f})/\mathbb{Z}_{N_f}$, broken to O($N_f$); but, since we are not worried about things like domain walls, it suffices to consider a connected component of the vacuum manifold.

In the infrared, the physical degrees of freedom are $N_f(N_f+1)/2-1$ massless Goldstone bosons, whose dynamics is described by an effective chiral lagrangian — a non-linear σ-model, whose target space is $\mathrm{SU}(N_f)/\,\mathrm{SO}(N_f)$. At least for small Goldstone boson fields, we can parametrize the coset as

$$U(x)=e^{i\pi^a(x)s_a/f_\pi}\,, \tag{1.2}$$

where the s_a are real, traceless, symmetric matrices, satisfying $\mathrm{tr}(s_a s_b)=2\delta_{ab}$. The most relevant term in the action contains two derivatives and may be written as

$$S_2[U]=-\frac{f_\pi^2}{4}\int d^4x\ \mathrm{Tr}\left(\frac{1}{2}\Big(U\partial U^{-1}+(U\partial U^{-1})^T\Big)\right)^2. \tag{1.3}$$

One can easily check that it is invariant under global right-$\mathrm{SU}(N_f)$ and *local* left-$\mathrm{SO}(N_f)$ transformations

$$U(x)\longrightarrow h(x)U(x)g^{-1}\,,\quad g\in \mathrm{SU}(N_f)\,,\ h(x)\in \mathrm{SO}(N_f)\,. \tag{1.4}$$

The dimensionful constant f_π is related to the strength of the chiral fermion condensate (1.1).

There is a one-loop chiral anomaly associated to the $\mathrm{SU}(N_f)$ symmetry group, whose manifestation at low energies is encoded by the Wess-Zumino action.[2] Using some extension of $U(x)$ to the 5-ball, the WZ action is

$$S_{WZ}[U]=-\frac{i(N^2-1)}{120\,\pi^2}\int_{B_5}\mathrm{Tr}\left(\frac{1}{2}\Big(UdU^{-1}+(UdU^{-1})^T\Big)\right)^5 \tag{1.5}$$

and is the leading term odd in the number of Goldstone boson fields. The normalization is *twice* what we naively expect for $U(x)\in \mathrm{SU}(N_f)$.[3] It is invariant under global right-$\mathrm{SU}(N_f)$ and *topologically-trivial* local left-$\mathrm{SO}(N_f)$ transformations. However $\pi_4(\mathrm{SO}(N_f)\neq 0$. The Wess-Zumino action shifts by $2\pi i$ under homotopically-non-trivial $\mathrm{SO}(N_f)$ transformations. Were it not for the extra factor of 2 in (1.5), $e^{-S_{WZ}[U]}$ would change sign under $\mathrm{SO}(N_f)$ transformations which are non-trivial elements of π_4.

We are interested in correlation functions of the $\mathrm{SU}(N_f)$ conserved currents, J. At low momenta (or widely separated points), these may be computed from the

[2] Here and below, we work in euclidean signature.

[3] The quantization of the WZ action is slightly subtle here. As we will mainly be interested in $N_f=3$, let us specialize to that case (similar results hold for $N_f=4,5$). The result (1.5) is twice what we would get by normalizing it to the integral generator of $\mathrm{H}^5(\mathrm{SU}(3))$. Consider the fiber bundle $\mathrm{SU}(3)\xrightarrow{\pi}\mathrm{SU}(3)/\,\mathrm{SO}(3)$, with fiber $\mathrm{SO}(3)$. We claim that if y is the generator of $\mathrm{H}^5(\mathrm{SU}(3)/\,\mathrm{SO}(3),\mathbb{Z})$, then π^*y is *twice* the generator of $\mathrm{H}^5(\mathrm{SU}(3),\mathbb{Z})$. We relegate the details to appendix B.

Goldstone boson effective lagrangian. The simplest way to study such correlators is to add sources A for the currents J to the two-derivative action (1.3),

$$S_2[U,A] = -\frac{f_\pi^2}{4}\int d^4x \ \mathrm{Tr}\left(\frac{1}{2}\Big(U(\partial+A)U^{-1} + \big(U(\partial+A)U^{-1}\big)^T\Big)\right)^2 \qquad (1.6)$$

in a way that now (1.6) is invariant under *local* $\mathrm{SU}(N_f)$ transformations, given by (1.4) and

$$A \longrightarrow gAg^{-1} + gdg^{-1}\,.$$

Similarly, one can gauge the Wess-Zumino term, $S_{WZ}[U,A]$, such that its variation under local $\mathrm{SU}(N_f)$ transformations reproduces the chiral anomalies of the underlying fermion theory (for a discussion of the general case of G/H, see [3]).

By differentiating with respect to A and setting $A=0$, we obtain the correlation functions of the currents. In momentum space, the two point function is

$$\left\langle J^a_\mu(p_1)J^b_\nu(p_2)\right\rangle \underset{p\to 0}{=} f_\pi^2\delta^{ab}\frac{p_\mu p_\nu}{p^2}(2\pi)^4\delta^{(4)}(p_1+p_2)\,. \qquad (1.7)$$

The three-point function contains the anomaly

$$\begin{aligned} p_1^\mu \left\langle J^a_\mu(p_1)J^b_\nu(p_2)J^c_\lambda(p_3)\right\rangle &= \\ &= -\frac{i}{2^8 3\pi^2}\left(N_c^2-1\right) d^{abc}\epsilon^{\nu\lambda\rho\sigma}p_{2\rho}p_{3\sigma}(2\pi)^4\delta^{(4)}(p_1+p_2+p_3)\,, \end{aligned} \qquad (1.8)$$

where $d^{abc} = \frac{1}{2}\mathrm{Tr}(s^a\{s^b,s^c\})$. So by studying the two- and three-point functions of the currents at low momenta, we can determine f_π.

For $N_f = 3$, this theory can be embedded in $\mathcal{N}=4$ SYM as follows. Starting with the $\mathcal{N}=4$ theory, we add a mass term for one of the gluinos,

$$\Delta L = \mathrm{tr}\left(m\lambda^4\lambda^4 + \overline{m}\overline{\lambda}_4\overline{\lambda}_4\right). \qquad (1.9)$$

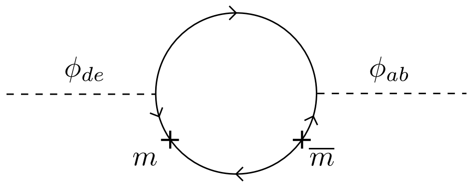
This breaks all the supersymmetries and breaks $\mathrm{SU}(4)_R$ to $\mathrm{SU}(3)$. Integrating out the massive fermion at one loop (exact, since the fermions appear only quadratically in the action), we obtain a common mass-squared for all six scalars in the $\mathcal{N}=4$ vector multiplet. To see this, it is convenient to write the **6** as the antisymmetric product of two **4**s. A convenient basis, ϕ_{ab} $(a,b=1,\ldots,4)$, for the scalars is $(I=1,2,3)$

$$\phi_{i4} = \Phi_I + i\Phi_{I+3}\,, \qquad \frac{1}{2}\epsilon^{ijk}\phi_{jk} = \Phi_I - i\Phi_{I+3}\,.$$

The Yukawa couplings are

$$\sqrt{2}\ \mathrm{tr}\Big(\phi_{ab}\left(\lambda^a\lambda^b + \epsilon^{abcd}\overline{\lambda}_c\overline{\lambda}_d\right)\Big).$$

The one-loop diagram with two insertions of the fermion mass matrix, m_{ab}, induces a scalar mass term proportional to



$$\sim \epsilon^{acde} (m\overline{m})_c{}^b \phi_{ab}\phi_{de}$$

which, for $m_{44} = m$ and otherwise zero, is just a common mass for all six scalars.

Thus, at low energies, we obtain a non-supersymmetric SU(N) gauge theory with three massless adjoint fermions and no scalars — in other words, adjoint QCD for $N_f = 3$. We use the $\mathcal{N} = 4$ theory as a particular ultraviolet regularization of this $N_f = 3$ theory. For large N, we can try to exploit the AdS/CFT correspondence to study it.

In [4, 5], it was observed that there is a supergravity solution which interpolates between the maximally supersymmetric SU(4)-invariant critical point of the 5D supergravity potential, located at the origin of the scalar field space, and the non-supersymmetric SU(3)-invariant critical point located at some point $\sigma = \sigma_\star \neq 0$ on the SU(3)-invariant direction, parametrized by σ, of the scalar field space. Both critical points have negative cosmological constant and correspond to anti-de Sitter spaces [6].

In light of the AdS/CFT correspondence [7, 8, 9], the conformal field associated to the supergravity field σ is the fermion mass term (1.9) and refs. [4, 5] interpreted the interpolating supergravity solution from $\sigma = 0$ to $\sigma_\star \neq 0$ as the renormalization group flow of the $\mathcal{N} = 4$ SU(N) gauge theory, at large-N and strong 't Hooft coupling, perturbed by the relevant perturbation (1.9).

If we believe in this supergravity solution, we would be led to expect that the boundary field theory flows from the $\mathcal{N} = 4$ superconformal fixed point in the UV to a non-supersymmetric conformal fixed point in the IR. This is in sharp contrast to the field theory expectations outlined above. Admittedly, the supergravity description is valid at large-N and strong 't Hooft coupling, where our grasp of the field theory is less secure. But still, a non-trivial infrared fixed point for this theory would be a surprise.

Anti-de Sitter spacetimes with scalar fields have to satisfy a particular lower bound for the masses of such scalars, the Breitenlohner-Freedman bound [10] (see also [11]), in order to have an (at least perturbatively) stable gravitational background under scalar fluctuations. Hence, in [4] we stressed the importance of performing an explicit check of the Breitenlohner-Freedman bound in non-supersymmetric gravitational backgrounds.[4] Among the models we discussed in [4] was an SU(3)-invariant five-dimensional anti-de Sitter background, which we found to be stable.

[4]If the background preserves some supersymmetries, the Breitenlohner-Freedman bound is guaranteed.

We now believe that calculation was in error, and the SU(3) critical point is unstable.[5] In the five-dimensional $\mathcal{N} = 8$ supergravity multiplet there are ten complex scalars in the irrep **10** of SU(4). Decomposing this representation under its SU(3) subgroup, $\mathbf{10} = \mathbf{1} + \mathbf{3} + \mathbf{6}$, we obtain the singlet, σ, a triplet which is higgsed for $\sigma \neq 0$ and six physical complex scalars in the symmetric tensor, $\square\!\square$, of SU(3). Upon correcting our previous calculation, it turns out that the scalars in the **6** are unstable modes, which violate the Breitenlohner-Freedman bound.[6] The tiniest fluctuations drive us away from the would-be SU(3) critical point.

What does this mean for the field theory? If the SU(3) critical point is unstable, the renormalization group flow of the SU(3) invariant theory certainly does not end up at any non-trivial infrared fixed point. But then, where does it flow to? To elucidate this question, first we should identify the conformal fields in the boundary theory which are associated to the **6**. They are the fermion bilinears

$$\mathcal{O}^{ij} = \mathrm{tr}\lambda^i\lambda^j \,.$$

From our previous discussion of chiral symmetry breaking in adjoint QCD, the most plausible interpretation is that, even though we do not couple $\mathcal{O}^{ij}$ to a source at the boundary — as we do for the conformal field which couples to σ — the renormalization group flow, read on the supergravity field space, is driven in a particular direction where some unstable modes in the **6** are turned on. That is, whereas the SU(4) symmetry is broken explicitly, the SU(3) symmetry is broken *dynamically* by an expectation value for the operator $\mathcal{O}^{ij}$.

The purpose of this paper is to examine these phenomena from the point of view of the five-dimensional supergravity and, in particular, to elucidate the fate of the unstable SU(3) critical point. One of the main goals is to search for Goldstone boson effects from the supergravity side. We will compute the "pion" decay constant f_π in this model of adjoint QCD from the two-point correlation function of the spontaneously broken currents J^μ.

2. The supergravity background

Under the SO(3) subgroup of SU(3) that leaves invariant the condensate (1.1), irreps of SU(3) have the following branching rules:

$$\mathbf{3} = \mathbf{3}\,, \qquad \mathbf{6} = \mathbf{5} + \mathbf{1}\,, \qquad \mathbf{8} = \mathbf{5} + \mathbf{3}\,. \tag{2.1}$$

[5] We would like to thank Freedman, Gubser, Pilch and Warner for pointing out to us that the stability analysis of the SU(3) point in [4] was probably incorrect. We are especially grateful to Krzysztof Pilch for a lengthy series of email exchanges in which we reconciled our respective calculations.

[6] In appendix A we write the scalar spectrum at the SU(3) critical point.

The supergravity field corresponding to the chiral condensate (1.1) is the SO(3) singlet in the **6** of SU(3). We parametrize this complex scalar as $\chi e^{i\beta}$. Besides this field, there are two additional complex scalars which are singlets of SO(3) in the 5D supergravity multiplet with non-zero boundary conditions. There is the SU(4)-invariant dilaton and axion, parametrized by the complex scalar $\rho e^{i\alpha}$, and the SU(3)-invariant complex field $\sigma e^{i\gamma}$, whose source term is the non-zero gluino mass m.[7]

2.1 Supergravity lagrangian and equations of motion

The next thing we need is the exact dependence of the $\mathcal{N} = 8$ five-dimensional supergravity lagrangian on these SO(3)-singlet scalars. Details of its parametrization and of the computation are contained in appendix A.[8] Here we just present the result:

$$
\begin{aligned}
\mathcal{L} = \frac{\sqrt{g}}{2\kappa_5^2}\bigg\{ & -\frac{1}{4}R + \frac{1}{2}(\partial\chi)^2 + \frac{1}{2}(\partial\sigma)^2 + \\
& + \frac{1}{4}\left(1 + \cosh^3\left(\frac{2\chi}{\sqrt{3}}\right)\cosh(2\sigma) - \right. \\
& \qquad \left. - \cos(2\alpha + 3\beta + \gamma)\sinh^3\left(\frac{2\chi}{\sqrt{3}}\right)\sinh(2\sigma)\right) \times \\
& \times \left((\partial\rho)^2 + \frac{1}{4}\sinh^2(2\rho)(\partial\alpha)^2\right) + \\
& + \frac{1}{2}\sin(2\alpha + 3\beta + \gamma)\sinh^3\left(\frac{2\chi}{\sqrt{3}}\right)\sinh(2\sigma)\sinh(2\rho)\,(\partial\rho)(\partial\alpha) + \\
& + \frac{3}{8}\sinh^2\left(\frac{2\chi}{\sqrt{3}}\right)\left(\partial\beta - \sinh^2(\rho)\,\partial\alpha\right)^2 + \\
& + \frac{1}{8}\sinh^2(2\sigma)\left(\partial\gamma + \sinh^2(\rho)\,\partial\alpha\right)^2 + V(\chi,\sigma)\bigg\}\,. \qquad (2.2)
\end{aligned}
$$

The potential only depends on the magnitudes χ and σ:

$$
\begin{aligned}
V(\chi,\sigma) = -\frac{3}{16L^2}\bigg\{ & 8 + \cosh\left(\frac{4\chi}{\sqrt{3}}\right) - \cosh(4\sigma) + \\
& + 4\cosh\left(\frac{2\chi}{\sqrt{3}} - 2\sigma\right) + 4\cosh\left(\frac{2\chi}{\sqrt{3}} + 2\sigma\right)\bigg\}\,. \qquad (2.3)
\end{aligned}
$$

[7]There is another SO(3) singlet in the **6** inside the $\mathbf{20'}$ of SU(4). But only the **6** in the **10** are unstable at the SU(3) critical point. Hence, in order to simplify our discussion, we will put that additional SO(3) singlet to zero.

[8]When this paper was being written, the preprint [12] appeared, where the supergravity lagrangian for the fields σ and χ (but not the dilaton or the phases) is also computed. Interestingly, they look for BPS solutions in supergravity where the role of the boundary conditions for σ and χ are interchanged with respect to ours. This simple switch gives completely different physics for the boundary field theory.

JHEP05(2000)005

If $2\alpha + 3\beta + \gamma = 0$, one can truncate the theory to the space of solutions with constant phases $\alpha, \beta, \gamma = 0$. This is not an accident. The effective CP-violating theta angle of the boundary theory turns out to be proportional to the combination of angles $2\alpha+3\beta+\gamma$. If it is non zero, the strong CP-violating phase starts to run under the renormalization group flow, with the flow triggered by the supergravity equations of motion. In order to focus on the mechanism of chiral symmetry breaking from supergravity in the simplest possible context, we will work in the sector of vanishing theta angle. In this sector, the equations of motion involve only the magnitudes χ, σ, ρ. Observe that the kinetic term of ρ is not canonical. For vanishing phases, it has the scalar-dependent coefficient

$$\mathcal{K}_{\rho\rho}(\chi, \sigma) = \frac{1}{4}\left\{\cosh^2\left(\sqrt{3}\chi - \sigma\right) + 3\cosh^2\left(\frac{\chi}{\sqrt{3}} + \sigma\right)\right\}. \tag{2.4}$$

Let us now write the supergravity equations of motion derived from (2.2). We parametrize the five-dimensional metric[9] by the Poincaré-invariant ansatz

$$ds^2 = e^{-2y(z)}\left(dz^2 + d\mathbf{x}^2\right). \tag{2.5}$$

For AdS_5 with radius L, we have $y(z) = \log(z/L)$, with the boundary located at $z = 0$. Then the equations of motion read:

$$\frac{d}{dz}\left(e^{-3y}\mathcal{K}_{\rho\rho}\,\dot{\rho}\right) = 0\,, \tag{2.6a}$$

$$\ddot{\chi} - 3\,\dot{y}\,\dot{\chi} - \frac{1}{2}(\partial_\chi \mathcal{K}_{\rho\rho})\,\dot{\rho}^2 - e^{-2y}(\partial_\chi V) = 0\,, \tag{2.6b}$$

$$\ddot{\sigma} - 3\,\dot{y}\,\dot{\sigma} - \frac{1}{2}(\partial_\sigma \mathcal{K}_{\rho\rho})\,\dot{\rho}^2 - e^{-2y}(\partial_\sigma V) = 0\,, \tag{2.6c}$$

$$6\,\dot{y}^2 - \dot{\chi}^2 - \dot{\sigma}^2 - \mathcal{K}_{\rho\rho}\,\dot{\rho}^2 + 2\,e^{-2y}\,V = 0\,, \tag{2.6d}$$

$$2\,\ddot{y} - \dot{\chi}^2 - \dot{\sigma}^2 - \mathcal{K}_{\rho\rho}\,\dot{\rho}^2 - \frac{2}{3}\,e^{-2y}\,V = 0\,. \tag{2.6e}$$

The last two equations are the Einstein's equations. It is easy to verify that (2.6e) is a consequence of the previous equations.

This coupled system of non-linear second order differential equations, plus the boundary conditions written below, codify the renormalization group flow of the $\mathcal{N} = 4$ SU(N) theory deformed by the SU(3)-invariant relevant coupling m, and the *possible* chiral symmetry breaking pattern SU(3) $\to$ SO(3).

2.2 Ultraviolet behavior

The ultraviolet theory is determined by the behavior of the supergravity solution for $z \ll L$. The boundary conditions for $z \to 0$ determine the lagrangian. In our case

[9] We will always work in the Einstein frame, and in euclidean signature.

they are

$$
\begin{aligned}
y &\longrightarrow \ln\left(\frac{z}{L}\right), && (2.7a)\\
\rho &\longrightarrow \rho_0\,, && (2.7b)\\
\sigma &\longrightarrow mz\,, && (2.7c)\\
\chi &\longrightarrow 0\,, && (2.7d)
\end{aligned}
$$

where by (2.7d) we mean that χ goes to zero faster than z. One can easily find the perturbative solution satisfying these boundary conditions:

$$
\begin{aligned}
y(z) &= \ln\left(\frac{z}{L}\right)\left(1+\frac{8}{15}m^4z^4+\mathcal{O}(z^6)\right)+\frac{1}{9}m^2z^2-\\
&\quad-\frac{m}{60}\left(\frac{524m^3}{135}-12\Lambda_\sigma^3\right)z^4+\mathcal{O}(z^6)\,, && (2.8a)\\
\sigma(z) &= mz\left(1+\frac{8}{3}m^2z^2\ln\left(\frac{z}{L}\right)+\frac{\Lambda_\sigma^3}{m}z^2+\frac{274}{45}m^4z^4\ln\left(\frac{z}{L}\right)+\mathcal{O}(z^4)\right), && (2.8b)\\
\chi(z) &= \Lambda_\chi^3z^3\Bigg(1-\frac{1}{6}m^2z^2+\frac{2}{45}m^4z^4\ln\left(\frac{z}{L}\right)+\\
&\qquad+\frac{m}{300}\left(5\Lambda_\sigma^3+14m^3\right)z^4+\mathcal{O}(z^6)\Bigg), && (2.8c)\\
\rho(z) &= \rho_0+\frac{\Lambda_\rho}{L^3}z^4\left(\frac{1}{4}-\frac{1}{9}m^2z^2+\mathcal{O}(z^4)\right). && (2.8d)
\end{aligned}
$$

Observe the appearance of the mass scales $\Lambda_\chi, \Lambda_\rho$ and Λ_σ. They arise because, for a given scalar with mass M^2 in anti-de Sitter space, there are two possible roots for the scaling dimension of the boundary conformal field to which it couples,

$$\Delta_\pm = 2 \pm \sqrt{4+M^2L^2}\,. \tag{2.9}$$

For the $\mathcal{N}=4$ fixed point, the appropriate branch is Δ_+. As we will see, the scales Λ, involving the Δ_- root, are related to the condensates of the corresponding operators[10] [13, 14].

2.3 Infrared behavior

Since the supergravity potential (2.3) does not have critical points when $\chi \neq 0$, the equations of motion (2.6) drive the fields χ and σ to infinity. This divergence usually produces a naked singularity, located at $z = z_0$, where the metric factor e^{-2y} vanishes.[11] This indicates that the field theory flows to a free phase in the infrared.

[10] As noted in [24], the correspondence between the branch Δ_- and the condensate has to be refined when the branch Δ_+ is also present in the supergravity solution for a given bulk field.

[11] Some references where naked singularities also appear in the study of supergravity duals are [15]–[21].

JHEP05(2000)005

Analysis of the equations of motion (2.6) near the singularity z_0 shows that there is a *unique* solution with the requisite behavior as $z \to z_0^-$. It is

$$y \simeq -\frac{3}{2}\log\left(\frac{z_0 - z}{L}\right) + \frac{3}{4}\log 42 - \frac{203}{4430}\left(\frac{z_0 - z}{L}\right)^2 + \cdots\,, \tag{2.10a}$$

$$\chi \simeq \sqrt{3}\left(-\frac{5}{4}\log\left(\frac{z_0 - z}{L}\right) + \frac{5}{8}\log 42 - \frac{167}{8860}\left(\frac{z_0 - z}{L}\right)^2 + \cdots\right), \tag{2.10b}$$

$$\sigma \simeq -\frac{1}{4}\log\left(\frac{z_0 - z}{L}\right) + \frac{1}{8}\log 42 - \frac{75}{1772}\left(\frac{z_0 - z}{L}\right)^2 + \cdots\,, \tag{2.10c}$$

$$\begin{aligned}\rho &\simeq \rho_0 + 2^{-1/4}\, 3^{-3/4}\, 7^{-1/4}\left(\frac{z_0 - z}{L}\right)^{7/2} \times \\ &\quad\times\left(\frac{1}{147} - \frac{23}{48730}\left(\frac{z_0 - z}{L}\right)^2 + \cdots\right).\end{aligned} \tag{2.10d}$$

If the SO(3)-symmetry is broken explicitly, there is also a continuous *family* of ultraviolet solutions, parametrized by the dimensionless ratios $m_\chi/m \neq 0$ and $\Lambda_\rho L$. Correspondingly, one expects a continuous family of infrared solutions. There is, indeed such a continuous family of infrared solutions. But, in addition to the continuous family, there is a single *isolated* solution, which is what we present in (2.10). Because it is isolated, we expect that it corresponds to the spontaneously broken case.

In the ultraviolet, our solution (2.8) had three undetermined dimensionless parameters, $\Lambda_{\chi,\rho,\sigma}/m$. The appearance of such undetermined parameters in the asymptotic solution near the boundary ($z = 0$) is typical; their values are determined by the behavior of the solution in the infrared.

That is what happens here. In principle, we could integrate our solution (2.10) numerically in the region $[0, z_0]$ and thereby determine the coefficients in (2.8). The remaining dimensionful parameter, m, corresponds to the location, z_0, of the singularity, which is determined by the integral of (2.6d).

We do not expect any further parameters, beyond N, $g_{\mathrm{YM}}^2 N$ and m, describing the field theory. So it is comforting that the complete supergravity solution does not have any. However, since we will not perform the numerical integration of the supergravity equations, our predictions for the field theory behavior will not be as powerful as they could be.

Since the supergravity potential is independent of the five-dimensional dilaton ρ, the dimensionful scale Λ_ρ enters as an integration constant. The infrared solution (2.10) determines its value to be

$$\Lambda_\rho = \frac{1}{16\sqrt{3}L}\,. \tag{2.11}$$

We cannot, in similar fashion, give the actual values of Λ_χ and Λ_σ without the explicit connection between the IR and the UV solution. But the fact that the IR solution has $\chi \neq 0$ *necessarily implies* that $\Lambda_\chi \neq 0$. We suspect that $\Lambda_\sigma = 0$.

2.4 The ten-dimensional geometry

In [22], a conjecture for the full ten-dimensional metric was given. If we parametrize the compact five-dimensional internal metric by $\{\theta^\alpha\}$ ($\alpha = 1, \ldots, 5$), the full metric is

$$ds_{10}^2 = \Delta^{-2/3} ds_5^2 + \tilde{g}_{\alpha\beta}\, d\theta^\alpha d\theta^\beta\,, \tag{2.12}$$

where $\tilde{g}_{\alpha\beta}$ is the five-dimensional internal metric. It is obtained from

$$\Delta^{-2/3}\tilde{g}^{\alpha\beta} = cK^\alpha{}_{IJ}K^\beta{}_{KL}(G^{-1})^{IJ,KL}\,, \tag{2.13}$$

where $\Delta = \sqrt{\det \tilde{g}_{\alpha\gamma}\tilde{g}_0{}^{\gamma\beta}}$, with $\tilde{g}_{0,\alpha\beta}$ the metric on the round S^5; $K^\alpha{}_{IJ} = -K^\alpha{}_{JI}$ are the 15 Killing vectors on the S^5; G^{-1} is the inverse matrix of the Sp(4) invariant matrix defined in (A.14); and c is a normalization constant.

We have computed the ten-dimensional metric in the SO(3)-invariant background (2.10). The result is complicated, and fairly unenlightening. What we do find, however, is that the full type-IIB geometry *is* singular at $z = z_0$.

3. Condensates and Goldstone bosons

3.1 Condensates

As was already observed in [13, 14], the dimensionful scales appearing in (2.8) are related to the condensates of the corresponding field theory operators. Let us first show this for the scale, Λ_ρ, associated to the dilaton. Take the dilaton term in the on-shell supergravity action and vary it with respect to the value of the background dilaton at $z = \epsilon$,[12]

$$\delta\left(\frac{1}{2\kappa_5^2}\int d^4x\, dz \sqrt{g}\frac{1}{2}\mathcal{K}_{\rho\rho}(\partial\rho)^2\right) = \frac{1}{2\kappa_5^2}\int d^4x \delta\rho(\epsilon)\left[e^{-3y}\mathcal{K}_{\rho\rho}\partial_z\rho\right]_{z=\epsilon}\,. \tag{3.1}$$

On the field theory side, this corresponds to taking the logarithmic derivative with respect to the square of the gauge coupling, defined at the ultraviolet cut-off length $z = \epsilon$.[13] We get

$$\left\langle \int d^4x \frac{1}{g_s}\mathrm{tr}F^2(x)\right\rangle = \delta^4(0)\frac{\Lambda_\rho}{\kappa_5^2}\,. \tag{3.2}$$

Taking into account the relation between the asymptotic five-dimensional gravitational constant and anti-de Sitter radius L,[14]

$$\frac{1}{2\kappa_5^2} = \frac{N^2}{2\pi^2 L^3} \tag{3.3}$$

[12] As usual, during the computation we need to regulate the theory by locating the boundary at $z = \epsilon$. Also, since the supergravity action is evaluated on-shell, only boundary terms contribute.

[13] In our parametrization of the five-dimensional dilaton $\rho e^{i\alpha}$, the string coupling is given by $g_s = e^\rho$ when $\alpha = 0$.

[14] Both are values at the boundary. Remember that the dilaton and the curvature depend on the bulk coordinate z.

and canceling the four-dimensional spacetime volume $\delta^4(0)$, we get

$$\left\langle \frac{1}{g_s}\mathrm{tr}F^2 \right\rangle = \frac{N^2\Lambda_\rho}{\pi^2 L^3} = \frac{N^2}{16\sqrt{3}\pi^2 L^4}\,. \tag{3.4}$$

Let us now compute the condensate of the SO(3)-invariant operator

$$\mathcal{O}_\chi = \frac{1}{\sqrt{3}}\sum_{i=1}^{3}\mathrm{tr}\lambda^i\lambda^i \tag{3.5}$$

associated to the bulk field χ. To compute it, we use the formula

$$\left\langle \int d^4x\, \mathcal{O}_\chi \right\rangle = -\frac{\partial}{\partial\chi_0}\langle e^{-\chi_0\int\mathcal{O}_\chi}\rangle\Big|_{\chi_0=0} = -\frac{\partial S_{\mathrm{sugra}}(\chi_0)}{\partial\chi_0}\Big|_{\chi_0=0}\,, \tag{3.6}$$

where χ_0 is a constant source for $\mathcal{O}_\chi$, which, after taking the derivative, we put to zero. That is, we evaluate it in the supergravity background corresponding to the SU(3)-invariant lagrangian. For $\chi_0 \neq 0$, the solution looks like

$$\chi(z) = \chi_0\, z + \Lambda_\chi^3\, z^3 + \mathcal{O}(\chi_0^2) \tag{3.7}$$

with corrections of $\mathcal{O}(\chi_0^2)$ to the remaining functions σ, ρ and y. Repeating the same manipulations as in the glueball condensate computation, we get

$$\langle\mathcal{O}_\chi\rangle = \frac{3N^2}{2\pi^2}\Lambda_\chi^3\,. \tag{3.8}$$

In the previous section, we observed that, due to the fact that the infrared solution has $\chi \neq 0$, the ultraviolet solution must have $\Lambda_\chi \neq 0$. So the chiral symmetry, SU(3), really is broken down to SO(3).

Note that both condensates have the expected field theory factor of N^2, as the fields are in the adjoint representation of the gauge group. Perhaps surprisingly, the glueball condensate (3.4) does not have any explicit m-dependence. So long as $m \neq 0$, its value is fixed by (2.11). On the other hand, we expect that Λ_χ is of order m.

3.2 Goldstone bosons

The boundary theory has a spontaneously broken global symmetry, so we expect to see Goldstone bosons. The spontaneous symmetry breaking of the global symmetry SU(3) to SO(3) produces 5 Goldstone bosons, which couple to the broken currents, as reviewed in the introduction. Let us see how their effects arise from the point of view of supergravity.

In the AdS/CFT correspondence, the $\mathrm{SU}(4)_R$ currents couple to gauge fields of the bulk-supergravity theory. If the gauge symmetry is unbroken, then the two-point function of the currents falls off as $1/|x|^6$. For the broken currents, however, we will

JHEP05(2000)005

find a slower falloff, $1/|x|^4$, as the distance, $|x|$, between the two insertions goes to infinity. This is the position-space version of (1.7),

$$\lim_{|x|\to\infty} \left\langle J_\mu{}^a(x) J_\nu{}^b(0) \right\rangle = \delta^{ab} f_\pi^2 \, \partial_\mu \partial_\nu \left(\frac{1}{|x|^2} \right) + \mathcal{O}\left(\frac{1}{|x|^6} \right) . \tag{3.9}$$

Our goal will be to extract the pion decay constant, f_π. Before we can do that, however, we need to normalize the currents correctly. To do this, we will choose them to be normalized so as to reproduce the anomaly (1.8). Happily, this computation has already been done (in the SU(4)-invariant AdS theory) in [23]. The anomaly is reproduced by the Chern-Simons term of the five-dimensional gauged supergravity action. It is unaffected by the perturbation we are doing.

Returning to the two-point function, we need the quadratic part of the action evaluated in the SO(3)-invariant background. For the gauge fields not in the unbroken SO(3) group, there is a mass term in the action generated by the non-zero profile of the fields σ and χ. In the unitary gauge corresponding to the background solution (2.8) and (2.10), the quadratic action for those gauge fields is

$$S = \frac{1}{2\kappa_5^2} \int dz \, d^4x \, \sqrt{g} \left(\frac{1}{2e^2} g^{ij} g^{kl} F^a_{ik} F^a_{jl} + \frac{1}{2} \left(\mathcal{M}^2\right)_{ab} g^{ij} A_i{}^a A_j{}^b \right) , \tag{3.10}$$

where $e^2 = 4/L^2$ is the five-dimensional SU(4) gauge coupling, $F_{ij} = \partial_i A_j - \partial_j A_i$ $(i,j = 1,\ldots,5;\ x^5 \equiv z)$. The mass matrix $\mathcal{M}^2{}_{ab}$ decomposes in two blocks. One, corresponding to the gauge fields not in the SU(3) subgroup, depends on both σ and χ, and the other, for the breaking of SU(3) to SO(3), depends only on the field χ. In terms of the supergravity theory, both have the same effect: they Higgs some gauge symmetries. But, as we will see below, those mass terms involving σ behave differently near the boundary and do not give rise to Goldstone bosons.

Let us focus on the gauge fields that point in the directions of the coset space SU(3)/ SO(3). It is easy to see that their mass matrix is proportional to the identity; hence we will not write the Lie algebra indices explicitly. We have to find the boundary-to-bulk propagator,

$$A_i(x,z) = \int d^4y \, K_{i\mu}(x-y,z) a_\mu(y) \tag{3.11}$$

with a_μ the source of the four-dimensional global current J^μ. The kernel $K_{i\mu}(x,z)$ has to satisfy the equations of motion coming from the action (3.10) and the boundary conditions

$$\lim_{z\to 0} K_{\nu\mu}(x,z) = \delta_{\nu\mu} \, \delta^4(x) \, , \qquad \lim_{z\to 0} K_{z\mu}(x,z) = 0 \, . \tag{3.12}$$

We do not know the exact expression for the kernel $K_{i\mu}(x,z)$ in our particular curved background because we only have a perturbative solution (2.8) to the supergravity equations. But we are only interested in the soft pion limit (3.9), and so the perturbative solution suffices, as all we need is an expansion of $K_{i\mu}$ in powers of mz.

The zeroth-order expression for the kernel, $K_{i\mu}^{(0)}$ is just the anti-de Sitter solution, but when $m \neq 0$ the kernel must satisfy the integrability condition

$$\partial_z K_{z\nu}^{(0)} + \partial_\mu K_{\mu\nu}^{(0)} = \frac{3}{z} K_{z\nu}^{(0)} . \tag{3.13}$$

This gives

$$\begin{aligned} K_{\mu\nu}^{(0)}(x,z) &= \frac{2z^2}{9\pi^2 |x|^6 (z^2+|x|^2)^3} \Big(\delta_{\mu\nu} |x|^2 \big(6|x|^4 + 3z^2|x|^2 + z^4\big) - \\ &\qquad\qquad - 4x_\mu x_\nu \big(3|x|^4 + 3z^2|x|^2 + z^4\big)\Big) , \\ K_{z\nu}^{(0)}(x,z) &= 0 \end{aligned} \tag{3.14}$$

which, as one can check, satisfies (3.12).

Let us now analyze the first-order perturbation in m. To do so, we will assume that $\Lambda_\chi \sim m$; i.e. they are of the same order. Due to the fact that the mass matrix is ${\mathcal{M}^2}_{ab} \sim \delta_{ab}\chi^2 \sim \delta_{ab}\Lambda_\chi^6 z^6$ near the boundary $z=0$, the first-order perturbation of the gauge boson equations of motion comes only from the metric

$$e^{-y(z)} \sim \frac{L}{z}\left(1 - \frac{1}{9}(mz)^2\right) . \tag{3.15}$$

There is a unique solution that does not have a pole[15] at $x=0$ for $z \neq 0$; it is

$$\begin{aligned} K_{\mu\nu}^{(1)}(x,z) &= \frac{(mz)^2}{81\pi^2}\left(\frac{4x_\mu x_\nu - \delta_{\mu\nu}|x|^2}{(z^2+|x|^2)^3} - \frac{3z^2\delta_{\mu\nu}}{(z^2+|x|^2)^3}\right) , \\ K_{z\nu}^{(1)}(x,z) &= 0 . \end{aligned} \tag{3.16}$$

The two-point function is given by

$$\langle J_\mu(x) J_\nu(y)\rangle = \frac{\delta^2}{\delta a^\mu(x) \delta a^\nu(y)} S[a]\Big|_{a=0} , \tag{3.17}$$

where the on-shell action becomes

$$\begin{aligned} S[a] &= \frac{1}{2\kappa_5^2 e^2} \int d^4x \, \left[e^{-y} A_\mu F_{z\mu}\right]_{z\to 0} = \\ &= \frac{N^2}{8\pi^2} \int d^4x \, d^4y \, a_\mu(x) \left[\frac{1}{z}\partial_z K_{\mu\nu}(x-y,z)\right]_{z\to 0} a_\nu(y) . \end{aligned} \tag{3.18}$$

If we plug in the solution $K_{i\mu} = K_{i\mu}^{(0)} + K_{i\mu}^{(1)}$, into (3.17) and take the limit $|x| \to \infty$, we obtain the expression (3.9), with

$$f_\pi^2 = \frac{N^2 m^2}{324\pi^4} . \tag{3.19}$$

Once again, we obtain the expected color factor, N^2.

[15] In our peculiar gauge choice, there is such a pole in the zero-th order kernel (3.14), but the singular term is pure-gauge in the five-dimensional sense. $K_{ij}^{(1)}$ is *not* pure-gauge in the five-dimensional sense, though $K_{\mu\nu}^{(1)}$ is pure-gauge in the four-dimensional sense.

JHEP05(2000)005

Note the, at first rather surprising, feature that we can read off the value of f_π (an infrared property of the field theory) from the behavior of the supergravity solution near the boundary. The solution for the propagator $K_{ij}(x,z)$ has an expansion in powers of mz. Since we are looking for the leading behavior in m, this turns out to involve the leading behavior in z, i.e. the behavior near the boundary. Physically, the bulk mode which dominates the two-point function has a z-dependent mass which vanishes near the boundary. For a large separation of points on the boundary, it prefers to propagate in a "skin" of depth $\sim 1/m$, rather than penetrating deep into the interior.

Finally, we turn to the currents which couple to the explicitly-broken generators in $\mathrm{SU}(4)/\mathrm{SU}(3)$. In this case, the gauge boson mass term depends of σ, giving $\mathcal{M}^2 \sim \sigma^2 \sim (mz)^2$ near the boundary. The mass term enters into the analysis of the perturbed equations of motion, and one finds that the two-point function of these currents decays exponentially with distance.

3.3 Wilson loops

Let us now discuss confinement in this theory. The effective five-dimensional $\mathcal{N}=8$ $\mathrm{SU}(4)$ gauged supergravity contains twelve two-forms $B^{I\alpha}$ ($I=1,\ldots,6$, $\alpha=1,2$) transforming as a $(\mathbf{6},\mathbf{2})$ of $\mathrm{SU}(4)\times\mathrm{SL}(2,\mathbb{R})$. They couple, in an $\mathrm{SL}(2,\mathbb{Z})$-invariant fashion, to the (p,q) type-IIB strings moving into the five-dimensional spacetime. The strength of this coupling is given by effective string tensions, $T_{\text{eff}}^{(p,q)}$, which in general are background dependent [19]. The values of these tensions are given by the square root of the eigenvalues corresponding to the $(\mathbf{6},\mathbf{2})$-block of the $\mathrm{Sp}(4)$-invariant metric

$$G_{I\alpha,J\beta} = V_{I\alpha}{}^{ab} V_{J\beta ab} f \tag{3.20}$$

that appears in the kinetic term of the two-forms

$$\epsilon_{\alpha\beta} B^{I\alpha}\wedge dB^{I\beta} + G_{I\alpha,J\beta} B^{I\alpha}\wedge \star B^{J\beta}\,. \tag{3.21}$$

The eigenvalues of $G_{I\alpha,J\beta}$ are functions of χ and σ; we give their values in the appendix A. At the origin of the field space $\{\chi,\sigma\}$, the eigenvalues are six copies of $\{e^{2\rho}, e^{-2\rho}\}$, corresponding to the fundamental and D1 strings, respectively. But when $\chi,\sigma\neq 0$, they are grouped in $\mathrm{SO}(3)$ triplets. From the point of view of the modified Wilson loops of Maldacena, this is to be expected. Their definition includes a term with the adjoint scalars X^I pointing in a particular direction of S^5. The six-dimensional vector representation of $\mathrm{SO}(6)$ decomposes into $\mathbf{3}+\mathbf{3}$ under our $\mathrm{SO}(3)$ embedding.

The static potential between a (p,q)-type dyon and its anti-dyon is given by the worldsheet that minimizes the action of the (p,q)-string in our background. For a separation r, we have

$$E_{(p,q)}(r) = \frac{1}{2\pi\alpha'} \int_{-r/2}^{r/2} dx\, e^{-2y}\, T_{\rm eff}^{(p,q)}(z) \sqrt{1 + \left(\frac{dz}{dx}\right)^2}\,. \tag{3.22}$$

The infrared behavior of the prefactor

$$e^{-2y} T_{\rm eff}^{(p,q)} \tag{3.23}$$

typically determines whether these dyons are confined. If it goes to zero, generically the charge is screened (a Higgs phase) If, however, if it goes to zero with a very particular power, we have a Coulomb phase [16, 19]. By contrast, if the positive quantity (3.23) does not go to zero, we typically have confinement. If it diverges in the infrared, with the absolute minimum at some point $z = z_{\rm min}$, then for large enough dyon-antidyon separation the (p, q) string is able to reach $z = z_{\rm min}$. The most energetically favorable situation is to remain there, giving a linear confining potential.

Things are more delicate when the minimum of (3.23) is at $z = z_0$. For large separation, the string connecting the dyons penetrates all the way to the singularity, where stringy corrections are important. This is what happens in our case. When $y \to \infty$, the two triplets of tensions go like[16]

$$T_{\rm eff}^{(1)} \sim \left\{ e^{2\frac{\chi}{\sqrt{3}}}, e^{-2\frac{\chi}{\sqrt{3}}} \right\}, \tag{3.24a}$$

$$T_{\rm eff}^{(2)} \sim \left\{ e^{\frac{\chi}{\sqrt{3}}+\sigma}, e^{-\frac{\chi}{\sqrt{3}}-\sigma} \right\}. \tag{3.24b}$$

For the SO(3)-invariant supergravity background we have found, the factor (3.23) goes to zero as $z \to z_0^-$. Hence, in order to conclude anything about the behavior of the Wilson loops, we need to understand the physics in the neighbourhood of the singularity.

4. Conclusions

We have studied chiral symmetry breaking in the context of the AdS/CFT correspondence. We made a relevant deformation of the $\mathcal{N} = 4$ SU(N) Yang Mills theory by the addition of a mass term for one of the fermions. We found a unique solution for the deformed supergravity equations of motion, which corresponds to a non-zero condensate of a quark bilinear in a singlet representation of the SO(3) subgroup of SU(3). We also computed the "pion" decay constant f_π of the Goldstone bosons from supergravity.

[16] Remember that the dilaton becomes irrelevant in this region.

Ideally, one would like to take $m \to \infty$, and thereby study the "pure" adjoint QCD theory with $N_f = 3$. This is possible, in principle, for *small* 't Hooft coupling. In that case, the scale at which the adjoint QCD theory becomes strong is far below the scale m, at which SU(4) is broken to SU(3). The supergravity, however, is valid for *large* 't Hooft coupling and, in this case, the chiral symmetry breaking scale is not all that different from the scale at which SU(4) is explicitly broken. To really probe the adjoint QCD theory (which is defined to be *weakly-coupled* at the cutoff scale m), we would need to study the full type-IIB string theory in this background. Alas, that is beyond the scope of the present work.

Acknowledgments

We would like to thank David Berenstein, Dan Freed, Willy Fischler, Vadim Kaplunovsky and Steven Weinberg for valuable discussions.

A. Computation of the supergravity lagrangian

The construction of the $D = 5$ $\mathcal{N} = 8$ SU(4) gauged supergravity lagrangian is explained in [6], and recently reviewed in [4, 24].

We are particularly interested in the scalar sector, which involves 42 scalars parametrizing the right coset space $\mathrm{Sp}(4) \setminus E_{6(6)}$. A point in this space is described by a 27×27 matrix $\mathcal{V}^{ab}{}_{AB}$. This 27-bein is in the $\overline{\mathbf{27}}$ of E_6 (acting on the E_6 indices $A, B = 1, \ldots, 8$ from the right) and $\mathbf{27}$ of Sp(4) (acting on the Sp(4) indices $a, b = 1, \ldots, 8$ from the left). It maps the $\mathbf{27}$ of $E_{6(6)}$ to the $\mathbf{27}$ of Sp(4).

To parametrize the scalar manifold, it is sometimes more convenient to work in a different basis for the $\mathbf{27}$ of $E_{6(6)}$. In the original ref. [6], the scalars were parametrized in an $\mathrm{SL}(6,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ real basis $\{Z_{IJ}, Z^{I\alpha}\}$ ($I, J = 1, \ldots, 6$ and $\alpha = 7, 8$ are $\mathrm{SL}(6,\mathbb{R})$ and $\mathrm{SL}(2,\mathbb{R})$ indices, respectively) for the $\mathbf{27}$ of $E_{6(6)}$, which decomposes into $(\overline{\mathbf{15}}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{2})$. This is useful because the gauged SO(6) embeds easily as a subgroup of $\mathrm{SL}(6,\mathbb{R})$.

In this basis, if we denote a generic element of the coset space $\mathrm{Sp}(4) \setminus E_{6(6)}$ by a real matrix $U = \exp(\Phi)$, the change of basis that relates U to the 27-bein (which is linked to basis of the local Sp(4) symmetries) is performed by 8×8 hermitian and pure imaginary matrices Γ_I that satisfy the SO(6) Clifford algebra,[17] and

$$\Gamma_0 = \pm i \prod_{I=1}^{6} \Gamma_I \,. \tag{A.1}$$

[17] [24] gives a particular realization of the SO(6) Clifford algebra.

The sign is determined by the particular complex structure chosen on the space on which $\mathrm{SL}(6,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ acts. The relation between $\mathcal{V}^{ab}{}_{AB}$ and $U^{AB}{}_{CD}$ is given by

$$\mathcal{V}^{ab}{}_{AB} = (\Gamma_{IJ})^{ab}\, U^{IJ}{}_{AB} + (\Gamma_{I\alpha})^{ab}\, U^{I\alpha}{}_{AB}\,, \tag{A.2}$$

where one defines

$$\Gamma_{IJ} = \frac{1}{2}(\Gamma_I\Gamma_J - \Gamma_J\Gamma_I)\,, \tag{A.3a}$$

$$\Gamma_{I\alpha} = (\Gamma_I, i\,\Gamma_I\Gamma_0)\,. \tag{A.3b}$$

To study the supergravity theory deformed by SU(3)-invariant boundary conditions, it proves convenient to construct the scalar manifold in an SU(3)-adapted basis. The construction was explained in [4], and we follow it here, except for a small re-ordering of the SU(3) irreps comprising the **27** of $E_{6(6)}$.[18] Here we take

$$\begin{pmatrix} \mathbf{1}_{(0,0)} \\ \mathbf{8}_{(0,0)} \\ \mathbf{3}_{(4,0)} \\ \overline{\mathbf{3}}_{(-4,0)} \\ \mathbf{3}_{(-2,2)} \\ \overline{\mathbf{3}}_{(2,2)} \\ \mathbf{3}_{(-2,-2)} \\ \overline{\mathbf{3}}_{(2,-2)} \end{pmatrix}. \tag{A.4}$$

The three SO(3) singlet complex scalars are introduced by the hermitian matrices (corresponding to non-compact generators)

$$\Phi_\rho = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&\rho e^{i\alpha}&0 \\ 0&0&0&0&0&0&0&\rho e^{i\alpha} \\ 0&0&0&0&\rho e^{-i\alpha}&0&0&0 \\ 0&0&0&0&0&\rho e^{-i\alpha}&0&0 \end{pmatrix}, \quad \Phi_\sigma = \begin{pmatrix} 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&\sigma e^{-i\gamma}&0 \\ 0&0&0&0&0&\sigma e^{i\gamma}&0&0 \\ 0&0&0&0&0&0&0&0 \\ 0&0&0&\sigma e^{-i\gamma}&0&0&0&0 \\ 0&0&\sigma e^{i\gamma}&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0 \end{pmatrix} \tag{A.5}$$

[18] To present a clearer change of basis matrix from $\mathrm{SL}(6,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$ to SU(3) basis.

and

$$\Phi_\chi = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{6}\, s^{\Upsilon} & 0 & 0 & \sqrt{6}\, \overline{s}^{\Upsilon} \\ 0 & 0 & 0 & 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \overline{s} & 0 \\ 0 & \sqrt{6}\, \overline{s}^{\Upsilon} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{s} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & s & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6}\, s^{\Upsilon} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.6}$$

where the bar means complex conjugation and ${\overline{s}^{\Upsilon}}^{ij}{}_k = \epsilon^{ijl}\, \overline{s}_{lk}$. For the SO(3) singlet, we have $s^{ij} = \sqrt{6} i \chi e^{i\beta}\, \delta^{ij}$.

To identify $\rho e^{i\alpha}$ with the complex flat direction of the potential, i.e. as the five-dimensional dilaton and axion, we parametrize the SO(3)-invariant submanifold[19] by

$$\mathcal{U}(\chi e^{i\beta}, \sigma e^{i\gamma}, \rho e^{i\alpha}) = e^{\Phi_\chi + \Phi_\sigma}\; e^{\Phi_\rho}\,. \tag{A.7}$$

To obtain the expressions of the 27-beins $\mathcal{V}^{ab}{}_{AB}$, we need the change of basis matrix that relates the SU(3) basis given in (A.4) to the real basis $\{Z_{IJ}, Z^{I\alpha}\}$. To relate them, we should first choose a complex structure on $\mathbb{R}^6$ and $\mathbb{R}^2$. We take, respectively,

$$\begin{aligned} z^i &= \frac{1}{\sqrt{2}}\left(x^i + i x^{i+3}\right), \quad i = 1,2,3\,, \\ u &= \frac{1}{\sqrt{2}}\left(x^7 + i x^8\right). \end{aligned} \tag{A.8}$$

This choice forces us to take the minus sign in (A.1).

Next we order the 27-dimensional real basis by the following nine triplets:

$$\left\{ Z^k_{(d)}, Z^k_{(s)}, Z^k_{(a)}, Z^k_{(0)}, Z^k_{(3)}, Z^{k,7}, Z^{k,8}, Z^{k+3,7}, Z^{k+3,8} \right\}$$

with the definitions:

$$\begin{aligned} &Z^k_{(d)} = Z_{k,k+3}\,, \qquad Z^1_{(s)} = \frac{1}{\sqrt{2}}(Z_{26} + Z_{35})\,, \qquad Z^2_{(s)} = \frac{1}{\sqrt{2}}(Z_{34} + Z_{16})\,, \\ &Z^3_{(s)} = \frac{1}{\sqrt{2}}(Z_{15} + Z_{24})\,, \qquad Z^1_{(a)} = \frac{1}{\sqrt{2}}(Z_{26} - Z_{35})\,, \qquad Z^2_{(a)} = \frac{1}{\sqrt{2}}(Z_{34} - Z_{16})\,, \\ &Z^3_{(a)} = \frac{1}{\sqrt{2}}(Z_{15} - Z_{24})\,, \qquad Z^k_{(0)} = \frac{1}{2}\epsilon^{kij} Z_{i,j}\,, \qquad Z^k_{(3)} = \frac{1}{2}\epsilon^{kij} Z_{i+3,j+3}\,. \end{aligned}$$

[19] Strictly speaking, this SO(3)-invariant submanifold has an additional dimension associated to a field in the **20′** of SO(6), which we put to zero in our discussion.

For the block $\mathbf{8}_{(\mathbf{0},\mathbf{0})}$ in (A.4), we take the ordering $\{H^1_{(d)}, H^2_{(d)}, H^k_{(s)}, H^k_{(a)}\}$ which, in the SU(3)-fundamental representation, acts by

$$H^i{}_j = \begin{pmatrix} H^1_{(d)} + \frac{H^2_{(d)}}{\sqrt{3}} & H^3_{(s)} + iH^3_{(a)} & H^2_{(s)} - iH^2_{(a)} \\ H^3_{(s)} - iH^3_{(a)} & -H^1_{(d)} + \frac{H^2_{(d)}}{\sqrt{3}} & H^1_{(s)} + iH^1_{(a)} \\ H^2_{(s)} + iH^2_{(a)} & H^1_{(s)} - iH^1_{(a)} & -\frac{2}{\sqrt{3}}H^2_{(d)} \end{pmatrix} . \tag{A.9}$$

Then, the unitary change of basis matrix, from the SL(6) × SL(2) to the SU(3) basis, is

$$M = \begin{pmatrix} N & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{2} & -\frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{2} & \frac{i}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & \frac{i}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{i}{2} & -\frac{i}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & \frac{i}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{i}{2} & -\frac{i}{2} & -\frac{1}{2} \end{pmatrix} , \tag{A.10}$$

where

$$N = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} \end{pmatrix}$$

is the orthogonal transformation that takes the diagonal entries $Z^k_{(d)}$ to the $\mathbf{1}_{(0,0)}$, $H^1_{(d)}$ and $H^2_{(d)}$.

We finally have all the ingredients to compute the SO(3)-singlet scalar sector of the supergravity lagrangian. The kinetic terms are obtained by projecting onto the non-compact generators of $E_{6(6)}$, which ensures invariance under the local left-action of Sp(4) on $\mathcal{U}$. Since compact generators are realized by antihermitian matrices, the projection is easily implemented by the formula

$$\mathcal{K} = -\frac{1}{48}\mathrm{tr}\left\{\left(\mathcal{U}\partial\mathcal{U}^{-1} + (\mathcal{U}\partial\mathcal{U}^{-1})^\dagger\right)^2\right\} . \tag{A.11}$$

The scalar potential is

$$V = -\frac{e^2}{32}\left(2W^{ab}W_{ab} - W^{abcd}W_{abcd}\right) \tag{A.12}$$

with

$$W^{abcd} = \epsilon^{\alpha\beta}\delta^{IJ}\mathcal{V}^{ab}{}_{I\alpha}\mathcal{V}^{cd}{}_{J\beta} , \qquad W^{ab} = W^c{}_{acb} \tag{A.13}$$

CFT operator	Supergravity field	$\mathrm{SU}(3)\times\mathrm{U}(1)$	Δ_{UV}	Δ_{IR}
$\lvert\mathrm{tr}(\lambda^4\lambda^4)\rvert$	σ	$\mathbf{1}_0$	3	$2+2\sqrt{3}=5.4641\cdots$
$\mathrm{tr}(\lambda^i\lambda^j)$	s^{ij}	$\mathbf{6}_8$	3	$2+i\frac{2}{3}$
$\mathrm{tr}(Z^iZ^j)$	t^{ij}	$\mathbf{6}_{-4}$	2	$2+\frac{2}{5}\sqrt{5}=2.8944\cdots$
$\mathrm{tr}(Z^i\overline{Z}_j-\frac{\delta^i{}_j}{3}\overline{\boldsymbol{Z}}\boldsymbol{Z})$	$h^i{}_j$	$\mathbf{8}_0$	2	4

Table 1: The scaling dimensions of the spinless conformal operators at the $SU(3)$ critical point.

and SU(4) gauge coupling $e = 2/L$ in order to have AdS_5 space with radius L at the origin of the field space. Assembling all the pieces, one obtains the lagrangian written in (2.2).

As in the tetrad formalism for General Relativity, one can define an Sp(4) invariant and $E_{6(6)}$ covariant metric form the 27-beins [25],

$$G_{AB,CD} = \mathcal{V}^{ab}{}_{AB}\Omega_{ac}\Omega_{bd}\mathcal{V}^{cd}{}_{CD}\,, \tag{A.14}$$

where Ω_{ab} is the Sp(4) symplectic metric. Since there is no quadratic invariant for $E_{6(6)}$, this metric should be used to construct the terms in the supergravity lagrangian that involve the vector and antisymmetric fields (except for the topological Chern-Simons terms). See [6] for the expressions of these terms.

To compute the Wilson loops from the five-dimensional supergravity theory, we will need the eigenvalues of the $(\mathbf{6},\mathbf{2})$-block $G_{I\alpha,J\beta}$. In the SO(3)-invariant background they split in two different SO(3) triplets. One is

$$\begin{aligned} T^{(1)}_{\text{eff}}{}^2 &= \frac{1}{2}\cosh(2\rho)\left(\cosh\left(\frac{4\chi}{\sqrt{3}}\right)+\cosh\left(\frac{2\chi}{\sqrt{3}}-2\sigma\right)\right)\pm \\ &\pm\frac{1}{2}\sqrt{\sinh^2(2\rho)\left(\cosh\left(\tfrac{4\chi}{\sqrt{3}}\right)+\cosh\left(\tfrac{2\chi}{\sqrt{3}}-2\sigma\right)\right)^2+\left(\cosh\left(\tfrac{4\chi}{\sqrt{3}}\right)-\cosh\left(\tfrac{2\chi}{\sqrt{3}}-2\sigma\right)\right)^2} \end{aligned} \tag{A.15}$$

and the other is

$$\begin{aligned} T^{(2)}_{\text{eff}}{}^2 &= \cosh(2\rho)\cosh^2\left(\frac{\chi}{\sqrt{3}}+\sigma\right)\pm \\ &\pm\sqrt{\sinh^2(2\rho)\cosh^4\left(\frac{\chi}{\sqrt{3}}+\sigma\right)+\sinh^4\left(\frac{\chi}{\sqrt{3}}+\sigma\right)}. \end{aligned} \tag{A.16}$$

Finally, in table 1 we correct table 1 of [4] for the scaling dimensions of the spinless conformal operators at the SU(3) critical point. Observe that the scaling dimension of the operator $\mathcal{O}^{ij} = \mathrm{tr}\lambda^i\lambda^j$ is complex, indicating that it violates the Breitenlohner-Freedman bound.

B. The Cohomology of $\mathrm{SU}(3)/\mathrm{SO}(3)$

The Wess-Zumino lagrangian is normalized to be the generator of $\mathrm{H}^5(\mathrm{SU}(3)/\mathrm{SO}(3), \mathbb{Z})$. We would like to write this in terms of its pullback to SU(3). To do that, we use the Leray Spectral Sequence (see [26, p. 169]) for the fiber bundle $\mathrm{SU}(3) \xrightarrow{\pi} \mathrm{SU}(3)/\mathrm{SO}(3)$.[20]

The cohomology ring of SU(3) is torsion-free, with generators in dimensions 3 and 5. That is,

Cohomology group	$\mathrm{H}^0(\mathrm{SU}(3),\mathbb{Z})$	$\mathrm{H}^3(\mathrm{SU}(3),\mathbb{Z})$	$\mathrm{H}^5(\mathrm{SU}(3),\mathbb{Z})$	$\mathrm{H}^8(\mathrm{SU}(3),\mathbb{Z})$
	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
generator	1	x_3	x_5	x_3x_5

The cohomology of SO(3) has torsion in H^2. The generator a satisfies $2a = 0$. Note that a is dual to the non-trivial 1-cycle in $\mathrm{H}_1(\mathrm{SO}(3),\mathbb{Z}) = \mathbb{Z}_2$ which originates because $\pi_1(\mathrm{SO}(3)) = \mathbb{Z}_2$.

Cohomology group	$\mathrm{H}^0(\mathrm{SO}(3),\mathbb{Z})$	$\mathrm{H}^2(\mathrm{SO}(3),\mathbb{Z})$	$\mathrm{H}^3(\mathrm{SO}(3),\mathbb{Z})$
	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}$
generator	1	a	x

From this data, we can compute the cohomology of $X = \mathrm{SU}(3)/\mathrm{SO}(3)$ (along with the information we are after) from the spectral sequence. To simplify the discussion, we just state the result for the cohomology of X, bearing in mind that it can be *derived* from the analysis below.

Cohomology group	$\mathrm{H}^0(X,\mathbb{Z})$	$\mathrm{H}^3(X,\mathbb{Z})$	$\mathrm{H}^5(X,\mathbb{Z})$
	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}$
generator	1	b	y

Again, the torsion element b, with $2b = 0$, arises because $\mathrm{H}_2(X,\mathbb{Z}) = \pi_2(X) = \pi_1(\mathrm{SO}(3)) = \mathbb{Z}_2$.

The Leray Spectral Sequence has as its E_2 term, $E_2^{p,q} = \mathrm{H}^p(X, \mathrm{H}^q(\mathrm{SO}(3)))$. These groups can be computed from the Universal Coefficients Theorem [26, p. 194],

$$\mathrm{H}^p(X,A) = \mathrm{Hom}(\mathrm{H}_p(X),A) \oplus \mathrm{Ext}(\mathrm{H}_{\mathrm{p}-1}(X),A)\,,$$

where

$$\mathrm{Ext}(\mathbb{Z}_m,\mathbb{Z}) = \mathbb{Z}_m\,, \qquad \mathrm{Ext}(\mathbb{Z}_m,\mathbb{Z}_n) = \mathbb{Z}_{gcd(m,n)}\,, \qquad \mathrm{Ext}(\mathbb{Z},\cdot) = 0\,.$$

The non-obvious cohomology group that one obtains is $\mathrm{H}^2(X, \mathrm{H}^2(\mathrm{SO}(3))) = \mathrm{Hom}(\mathbb{Z}_2, \mathbb{Z}_2) = \mathbb{Z}_2$. We call its generator c.

There is the differential $d_r : E_r^{p,q} \to E_r^{p-r+1,q+r}$, and to obtain the E_{r+1} term of the spectral sequence, we are instructed to take the cohomology of d_r.

[20] We would like to thank Dan Freed for helping us through this computation.

$$E_2^{p,q}\quad \begin{tikzcd}[row sep=small, column sep=small]
3 & x \arrow[drr, "d_2"] & & & xb \arrow[drr, "d_2"] & & xy \\
2 & a & & c & ab & & ay \\
1 & & & & & & \\
0 & 1 & & & b & & y \\
 & 0 & 1 & 2 & 3 & 4 & 5
\end{tikzcd}$$

For the E_2 term, we see that $d_2(xb) = ay$ and $d_2(x) = c$. Now, both xb and ay are of order 2. So, when we take the cohomology, they both get killed. However, x has infinite order, while $2c = 0$. So $d_2(2x) = 0$, and hence $2x$ survives in the cohomology.

$$E_3^{p,q}\quad \begin{tikzcd}[row sep=small, column sep=small]
3 & 2x & & & & & xy \\
2 & a \arrow[ddrrr, "d_3"] & & & ab & & \\
1 & & & & & & \\
0 & 1 & & & b & & y \\
 & 0 & 1 & 2 & 3 & 4 & 5
\end{tikzcd}$$

For the E_3 term, $d_3(a) = b$, and the spectral sequence converges at the E_4 term.

$$E_4^{p,q} = E_\infty^{p,q}\quad \begin{array}{c|cccccc}
3 & 2x & & & & & xy \\
2 & & & & ab & & \\
1 & & & & & & \\
0 & 1 & & & & & y \\
\hline
 & 0 & 1 & 2 & 3 & 4 & 5
\end{array}$$

Finally, we have

$$G^{(n)} = \bigoplus_{p+q=n} E_\infty^{p,q} = \mathrm{Gr}\left(\mathrm{H}^n(\mathrm{SU}(3))\right).$$

That is, $\mathrm{H}^n(\mathrm{SU}(3))$ has a filtration

$$\mathrm{H}^n(\mathrm{SU}(3)) = F_0 \supset F_1 \supset F_2 \supset F_3 \supset \cdots$$

such that $F_i/F_{i+1} = E_\infty^{i,n-i}$. The non-trivial case here is $\mathrm{H}^5(\mathrm{SU}(3)) = \mathbb{Z}$. We have $F_2/F_3 = \mathbb{Z}_2$ and $F_5/F_6 = \mathbb{Z}$. This means that $F_i = 0$ for $i \geq 6$ and $F_i = \mathbb{Z}$ for $i = 0, \ldots, 5$. The inclusions $F_i \hookrightarrow F_{i-1}$ for $i < 6$ are all isomorphisms, *except* for the map $F_3 \hookrightarrow F_2$, which is multiplication by two. The net effect is that the map

$$\pi^* : \mathrm{H}^5(\mathrm{SU}(3)/\mathrm{SO}(3), \mathbb{Z}) \longrightarrow \mathrm{H}^5(\mathrm{SU}(3), \mathbb{Z})$$

is multiplication by *two*.

This factor of two resolves an apparent puzzle [27]. Since $\pi_4(\mathrm{SO}(3)) = \mathbb{Z}_2$, there is an ambiguity in lifting the σ-model map into $\mathrm{SU}(3)/\mathrm{SO}(3)$ to a map into $\mathrm{SU}(3)$. Let w be the generator of $\pi_4(\mathrm{SO}(3))$. Then both $U(x)$ and $\tilde{U}(x) = w(x)U(x)$ represent the *same* sigma model map into the coset space $\mathrm{SU}(3)/\mathrm{SO}(3)$. In particular, we must have $e^{-S_{WZ}[\tilde{U}]} = e^{-S_{WZ}[U]}$. With the naive normalization of S_{WZ} coming from the generator of $\mathrm{H}^5(\mathrm{SU}(3), \mathbb{Z})$, we would instead find

$$e^{-S_{WZ}[w]} = -1\,. \tag{B.1}$$

But, from the previous analysis, we find that the correct normalization of S_{WZ}, coming from the generator of $\mathrm{H}^5(\mathrm{SU}(3)/\,\mathrm{SO}(3), \mathbb{Z})$, is *twice* the naive normalization, and thus $e^{-S_{WZ}[w]} = 1$, as required.

Similar computations can be carried out in the case of four flavours, where $\pi_4(\mathrm{SO}(4)) = \mathbb{Z}_2 + \mathbb{Z}_2$ and five flavours, where $\pi_4(\mathrm{SO}(5)) = \mathbb{Z}_2$. Indeed, the sign in (B.1) is intimately related to the global anomaly [28] in gauging H. So long as the underlying fermion theory is anomaly-free, the normalization of S_{WZ} must be such as to eliminate this sign.

References

[1] S. Coleman and E. Witten, *Chiral symmetry breakdown in large-N chromodynamics*, *Phys. Rev. Lett.* **45** (1980) 100.

[2] C. Vafa and E. Witten, *Restrictions on symmetry breaking in vector-like gauge theories*, *Nucl. Phys.* **B 234** (1984) 173.

[3] C.-S. Chu, P.-M. Ho and B. Zumino, *Non-abelian anomalies and effective actions for a homogeneous space g/h*, *Nucl. Phys.* **B 475** (1996) 484 [hep-th/9602093].

[4] J. Distler and F. Zamora, *Non-supersymmetric conformal field theories from stable anti-de Sitter spaces*, *Adv. Theor. Math. Phys.* **2** (1999) 1405 [hep-th/9810206].

[5] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of N = 4 super Yang-Mills from AdS dynamics*, *J. High Energy Phys.* **12** (1998) 022 [hep-th/9810126].

[6] M. Gunaydin, L. J. Romans, and N. P. Warner, *Compact and noncompact gauged supergravity theories in five-dimensions*, *Nucl. Phys.* **B 272** (1986) 598.

[7] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[8] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[9] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[10] P. Breitenlohner and D. Z. Freedman, *Stability in gauged extended supergravity*, *Ann. Phys. (NY)* **144** (1982) 249.

[11] L. Mezincescu and P. K. Townsend, *Stability at a local maximum in higher dimensional anti-de Sitter space and applications to supergravity*, *Ann. Phys. (NY)* **160** (1985) 406.

[12] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of* $N = 1$ *super Yang-Mills theory*, *Nucl. Phys.* **B 569** (2000) 451 [hep-th/9909047].

[13] V. Balasubramanian, P. Kraus, A. Lawrence and S.P. Trivedi, *Holographic probes of anti-de Sitter space-times*, *Phys. Rev.* **D 59** (1999) 104021 [hep-th/9808017].

[14] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[15] S. Nojiri and S.D. Odintsov, *Two-boundaries AdS/CFT correspondence in dilatonic gravity*, *Phys. Lett.* **B 449** (1999) 39 [hep-th/9812017].

[16] J.A. Minahan, *Asymptotic freedom and confinement from type-0 string theory*, *J. High Energy Phys.* **04** (1999) 007 [hep-th/9902074].

[17] A. Kehagias and K. Sfetsos, *On running couplings in gauge theories from type-IIB supergravity*, *Phys. Lett.* **B 454** (1999) 270 [hep-th/9902125].

[18] S.S. Gubser, *Dilaton-driven confinement*, hep-th/9902155.

[19] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Confinement and condensates without fine tuning in supergravity duals of gauge theories*, *J. High Energy Phys.* **05** (1999) 026 [hep-th/9903026].

[20] A. Kehagias and K. Sfetsos, *On asymptotic freedom and confinement from type-IIB supergravity*, *Phys. Lett.* **B 456** (1999) 22 [hep-th/9903109].

[21] N.R. Constable and R.C. Myers, *Exotic scalar states in the AdS/CFT correspondence*, *J. High Energy Phys.* **11** (1999) 020 [hep-th/9905081].

[22] A. Khavaev, K. Pilch and N.P. Warner, *New vacua of gauged* $N = 8$ *supergravity in five dimensions*, hep-th/9812035.

[23] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the* CFT_d/AdS_{d+1} *correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058].

[24] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, hep-th/9904017.

[25] E. Cremmer, *Supergravities in 5 dimensions*, in *Proceedings of the nuffield gravity workshop*, Cambridge, England, June 22 – July 12, 1980.

[26] R. Bott and L. Tu, *Differential forms in algebraic topology.* Springer-Verlag, 1982

[27] E. D'Hoker and S. Weinberg, *General effective actions*, *Phys. Rev.* **D 50** (1994) 6050 [hep-ph/9409402].

[28] E. Witten, *An* SU(2) *anomaly*, *Phys. Lett.* **B 117** (1982) 324.